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Broj: 2023/01-1069/1
Datum: 11.06.2023.

UNIVERZITET CRNE GORE

SENATU

CENTRU ZA DOKTORSKE STUDIJE

U prilogu akta dostavljamo Odluku sa C sjednice Vijeća Prirodno-matematičkog fakulteta održane 13.06.2023. godine.

S poštovanjem,

UNIVERZITET CRNE GORE Prirodno-matematički fakultet UCG

Dekan,

Prof. dr Miljan Bigović

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Broj: 1071/01-1069/3
Datum: 15. 06. 2023

Na osnovu člana 64 Statuta Univerziteta a u vezi sa članom 41 stav 1 Pravila doktorskih studija, Predloga prof. dr Davida Kaljaja broj 2023/01-1069 od 26.5.2023.godine, na C sjednici Vijeća održanoj dana 13.6. 2023. godine, utvrđen je

PREDLOG ODLUKE

I

Utvrđuje se da su ispunjeni uslovi iz člana 38 Pravila doktorskih studija.

II

Predlaže se komisija za ocjenu doktorske disertacije MSc Antona Đokaja u sljedećem sastavu:

1. Prof. dr Darko Mitrović, redovni profesor na PMF-u UCG
2. Prof. dr David Kaljaj, redovni profesor na PMF-u UCG;
3. Prof. dr Miodrag Matejević, redovni profesor Matematičkog fakulteta Univerziteta u Beogradu;
4. Prof. dr Marijan Marković, vanredni profesor PMF-a UCG i
5. Prof. dr Đorđije Vujadinović, vanredni profesor na PMF-u UCG.

III

Odluka se dostavlja Odboru za doktorske studije na dalje postupanje.

DEKAN
Prof. dr Miljan Bigović

ISPUNJENOST USLOVA DOKTORANDA

OPŠTI PODACI O DOKTORANDU			
Titula, ime, ime roditelja, prezime	MSc Anton, Gjelosh, Gjokaj		
Fakultet	Prirodno-matematički fakultet		
Studijski program	Matematika		
Broj indeksa	2/19		
NAZIV DOKTORSKE DISERTACIJE			
Na službenom jeziku	Granična svojstva kvazikonformnih harmonijskih preslikavanja u prostoru		
Na engleskom jeziku	Boundary behaviour of quasiconformal harmonic mappings in space		
Naučna oblast	Matematika		
MENTOR/MENTORI			
Prvi mentor	Prof. dr David Kalaj	Prirodno-matematički fakultet, Univerzitet Crne Gore, Crna Gora	Matematička analiza
KOMISIJA ZA PREGLED I OCJENU DOKTORSKE DISERTACIJE			
Prof. dr Darko Mitrović		Prirodno-matematički fakultet, Univerzitet Crne Gore, Crna Gora	Matematička analiza
Prof. dr David Kalaj		Prirodno-matematički fakultet, Univerzitet Crne Gore, Crna Gora	Matematička analiza
Prof. dr Miodrag Mateljević		Matematički fakultet, Univerzitet u Beogradu, Srbija	Matematička analiza
Prof. dr Marijan Marković		Prirodno-matematički fakultet, Univerzitet Crne Gore, Crna Gora	Matematička analiza
Prof. dr Đorđije Vujadinović		Prirodno-matematički fakultet, Univerzitet Crne Gore, Crna Gora	Matematička analiza
Datum značajni za ocjenu doktorske disertacije			
Sjednica Senata na kojoj je data saglasnost na ocjenu teme i kandidata	15.04.2022. godine		
Dostavljanja doktorske disertacije organizacionoj jedinici i saglasanost mentora	24.05.2023. godine		
Sjednica Vijeća organizacione jedinice na kojoj je dat prijedlog za imenovanje komisija za pregled i ocjenu doktorske disertacije	13. 6. 2023. g.		
ISPUNJENOST USLOVA DOKTORANDA			

U skladu sa članom 38 pravila doktorskih studija kandidat je cjelokupna ili dio sopstvenih istraživanja vezanih za doktorsku disertaciju publikovao u časopisu sa **(SCI/SCIE)/(SSCI/A&HCI)** liste kao prvi autor.

Spisak radova doktoranda iz oblasti doktorskih studija koje je publikovao u časopisima sa (upisati odgovarajuću listu)

1. Anton Gjokaj, Hölder continuity of quasiconformal harmonic mappings from the unit ball to a spatial domain with C^1 boundary, Indagationes Mathematicae, Volume 33, Issue 5, 2022, Pages 1061-1070, ISSN 0019-3577, https://doi.org/10.1016/j.indag.2022.05.003

2. A. Gjokaj, D. Kalaj, Quasiconformal harmonic mappings between the unit Ball and a spatial domain with $C^{1,\alpha}$ Boundary, Potential Analysis, Volume 57, 367-377 (2022). https://doi.org/10.1007/s11118-021-09919-y

Obrazloženje mentora o korišćenju doktorske disertacije u publikovanim radovima

U radu objavljenom u časopisu *Ingadationes Mathematicae* kandidat je dokazao uniformnu Hölder neprekidnost za kvazikonformna harmonijska Bloch preslikavanja f iz jedinične lopte $B \subset R^n$ u prostornu oblast sa C^1 granicom.

U radu objavljenom u časopisu *Potential Analysis* dokazana je Lipschitz neprekidnost kvazikonformnih harmonijskih preslikavanja iz jedinične lopte B u prostornu oblast sa $C^{1,\alpha}$ granicom. U ovom radu su date i dvije verzije Hardy-Littlewood teoreme za prostor, kod koje se daje veza između μ-Hölder koeficijenta ($\mu < 1$) u odnosu na tačku $\eta \in S$, tj.

$$\sup_{\xi \in S, \xi \neq \eta} \frac{\|u(\eta) - u(\xi)\|}{\|\eta - \xi\|^\mu}$$

i vrijednosti

$$\sup_{x \in [0,\eta)} (1 - \|x\|)^{1-\mu} \|\nabla u(x)\|.$$

Datum i ovjera (pečat i potpis odgovorne osobe)		
U Podgorici, 13. 6. 2023.	MP	DEKAN

Prilog dokumenta sadrži:

1. Potvrdu o predaji doktorske disertacije organizacionoj jedinici
2. Odluku o imenovanju komisije za pregled i ocjenu doktorske disertacije
3. Kopiju rada publikovanog u časopisu sa odgovarajuće liste
4. Biografiju i bibliografiju kandidata
5. Biografiju i bibliografiju članova komisije za pregled i ocjenu doktorske disertacije sa potvrdom o izboru u odgovarajuće akademsko zvanje i potvrdom da barem jedan član komisije nije u radnom odnosu na Univerzitetu Crne Gore

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Broj: 2023/01-1069/3

Datum: 26.05.2023. god

Na osnovu člana 33 Zakona o upravnom postupku, nakon uvida u službenu evidenciju, Prirodno-matematički fakultet izdaje

POTVRDU

MSc Anton Đokaj, student doktorskih studija na Prirodno-matematičkom fakultetu u Podgorici, dana 26.05.2023. godine, dostavio je ovom fakultetu doktorsku disetaciju pod nazivom ***„GRANIČNA SVOJSTVA KVAZIKONFORMNIH HARMONIJSKIH PRESLIKAVANJA U PROSTORU"*** na dalje postupanje.

Dekan,

Prof. dr Miljan Bigović

Univerzitet Crne Gore Prirodno-matematički fakultet europass

BIOGRAFIJA - CV

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RADNO ISKUSTVO

09/2017-trenutno **Saradnik u nastavi**

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Spisak predmeta:

- Analiza 3 (09/2017 - trenutno)
- Analiza 4 (09/2017 - trenutno)
- Kompleksna analiza 1 (09/2017 - trenutno)
- Kompleksna analiza 2 (09/2020 - trenutno)
- Diferencijalne jednačine (09/2017 - trenutno)
- Uvod u diferencijalnu geometriju (09/2017- trenutno)
- Algebra 1 (09/2017 - trenutno)
- Algebra (09/2017 – 09/2022)
- Matematika III -Elektrotehnički fakultet (09/2020 - trenutno)
- Matematika I - Mašinski fakultet (09/2017- 09/2020)

OBRAZOVANJE I OSPOSOBLJAVANJE

09/2019- trenutno **Doktorske studije**

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Položeni doktorski predmeti (indeks uspjeha 10.00)
Odbrana polaznih istraživanja doktorske disertacije: "Granična svojstva kvazikonformnih harmonijskih preslikavanja u prostoru" 03/2022

09/2017-03/2019 **Magistar (MSc) Matematike**

Prirodno-matematički fakultet, Univerzitet Crne Gore
Indeks uspjeha: 10.00
Master rad: „Konveksne i univalentne harmonijske funkcije u kompleksnoj ravni i njihova geometrija"

09/2016-07/2017 **Specijalista (Spec. Sci) Matematike**

Prirodno-matematički fakultet, Univerzitet Crne Gore
Indeks uspjeha: 10.00
Specijalistički rad: „Konformna preslikavanja između jednostruko i dvostruko povezanih oblasti"

09/2013-06/2016 **Bachelor (BSc) Matematike**

Prirodno-matematički fakultet, Univerzitet Crne Gore
Indeks uspjeha: 10.00

LIČNE VJEŠTINE

Maternji jezik: albanski

Ostali jezici

	RAZUMIJEVANJE		GOVOR		PISANJE
	Slušanje	Čitanje	Govorna interakcija	Govorna produkcija	
crnogorski	C2	C2	C2	C2	C2
engleski	B2	B2	B2	B2	B2
italijanski	B1	B1	B1	B1	B1

Nivoi: A1/2: Elementarna upotreba jezika - B1/B2: Samostalna upotreba jezika- C1/C2 Kompetentna upotreba jezika

Digitalna kompetencija

SAMOPROCJENA				
Obrada informacija	Komunikacija	Stvaranje sadržaja	Sigurnost	Rješavanje problema
Samostalna upotreba	Napredna upotreba	Samostalna upotreba	Samostalna upotreba	Samostalna upotreba

Nivoi: Elementarna upotreba - Samostalna upotreba - Kompetentna upotreba

DODATNE INFORMACIJE

Publikovani radovi

1. Anton Gjokaj, Hölder continuity of quasiconformal harmonic mappings from the unit ball to a spatial domain with C^1 boundary, Indagationes Mathematicae, Volume 33, Issue 5, 2022, Pages 1061-1070, ISSN 0019-3577,https://doi.org/10.1016/j.indag.2022.05.003
2. A. Gjokaj, D. Kalaj, Quasiconformal harmonic mappings between the unit Ball and a spatial domain with $C^{1,\alpha}$ Boundary, Potential analysis, Volume 57, 367-377 (2022). https://doi.org/10.1007/s11118-021-09919-y

Prezentacije

- Zimska škola nauke Prona 2013, "Teorija igara. Problem vozača ubice", kao učenik
- Ljetnja škola nauke Prona 2021, "Beskonačni skupovi", kao pozvani predavač

Konferencije

- XII simpozijum "Matematika i primene", Beograd
  „Lipschitz continuity of quasiconformal mappings with L^p Laplacian, p>n, between the unit ball B^n and a spatial domain with $C^{1,\alpha}$", 12/2022

Seminari / Kursevi

- Radboud summer school, Nijmegen, 2019
- DAAD intensive course on "Measures of Noncompactness and Applications", Priština, 2014.godine

Priznanja i nagrade

- Nagrada Ministarstva nauke i tehnološkog razvoja za najuspješnijeg mladog naučnika do 30 godina života, 2022. godine
- Stipendija CANU najboljim studentima za rezultate na studijama, 2017. godine
- Univerzitetska plaketa, 2017. godine
- Univerzitetska nagrada, 2015. godine
- Olimpijada znanja 2013, Matematika, 4. godina – I mjesto (III nagrada[i])
- Državno takmičenje 2013, Matematika, srednja škola – II mjesto (II nagrada)
- Olimpijada znanja 2012, Biologija, 3. godina – III mjesto (III nagrada)
- Olimpijada znanja 2011, Matematika, 2. godina – I mjesto (II nagrada)
- Državno takmičenje 2009, Matematika, 9. razred – II mjesto (II nagrada)

Učesnik

- Internacionalna matematička olimpijada 2013, Santa Marta
- Balkanska matematička olimpijada 2013, Agros
- Juniorska balkanska matematička olimpijada 2009, Sarajevo

Časovi

- Spremanje učenika srednjih škola za Balkanska i internacionalna takmičenja iz matematike, 2020-trenutno

Član

- Komisija za sastavljanje i pregledanje zadataka na Državnom takmičenju iz Matematike za srednju školu, 2020-trenutno

Certifikati

- ECDL Start Certificate Module 2,3,4,7

[i] Nagrade se dodjeljuju na osnovu osvojenih poena, dok mjesto predstavlja poziciju na konačnoj bodovnoj listi

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Quasiconformal Harmonic Mappings Between the Unit Ball and a Spatial Domain with $C^{1,\alpha}$ Boundary

Anton Gjokaj[1] · David Kalaj[1]

Received: 17 June 2020 / Accepted: 2 March 2021 / Published online: 26 March 2021

Abstract
We prove the following. If f is a harmonic quasiconformal mapping between the unit ball in $\mathbb{R}^n$ and a spatial domain with $C^{1,\alpha}$ boundary, then f is Lipschitz continuous in B. This generalizes some known results for $n = 2$ and improves some others in higher dimensional case.

Keywords Harmonic mappings · Quasiconformal mappings · Hölder continuity · Lipschitz continuity

Mathematics Subject Classification (2010) Primary 30C65 · Secondary 31B05

1 Introduction

For $n > 1$, let $\mathbb{R}^n$ be the standard Euclidean space with the norm $|x| = \left(x_1^2 + \ldots + x_n^2\right)^{\frac{1}{2}}$, where $x = (x_1, \ldots, x_n)$. We denote the unit ball $\{x \in \mathbb{R}^n : |x| < 1\}$ by B, and its boundary, the unit sphere $\{x \in \mathbb{R}^n : |x| = 1\}$ by S.

Let $U \subset \mathbb{R}^n$ be a domain. We say $f = (f_1, \ldots, f_n) : U \to \mathbb{R}^n$ is a harmonic mapping if the functions f_j are harmonic real mappings, i.e. satisfy the n-dimensional Laplace equation

$$\Delta f_j = \sum_{i=1}^{n} D_{ii} f_j = 0.$$

Let

$$P(x, \xi) = \frac{1 - |x|^2}{|x - \xi|^n}$$

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be the Poisson kernel for B, where $x \in B$, $\xi \in S$, and

$$P[u](x) = \int_S P(x,\xi)u(\xi)d\sigma(\xi)$$

the Poisson integral of continuous function u on S, where σ denotes the normalized surface-area measure on S. Then $P[u](x)$ is continuous on $\overline{B}$ and harmonic on B. Since we will focus on continuous function u on $\overline{B}$, that are harmonic on B, then we will usually express them using the Poisson integral as

$$u = P[u|_S](x).$$

A homeomorphism $f : U \to V$, where U, V are domains in $\mathbb{R}^n$, will be called K quasiconformal (see [27]) ($K \geq 1$) if f is absolutely continuous on lines (i.e. absolutely continuous in almost every segment parallel to some of the coordinate axes and there exist partial derivatives which are locally L^n integrable in U) and

$$|\nabla f(x)| \leq Kl(\nabla f(x)),$$

for all points $x \in U$, where

$$l(\nabla f(x)) = \inf\left\{|f'(x)h| : |h| = 1\right\}.$$

A function $\Phi : U \subset \mathbb{R}^n \to \mathbb{R}$ is said to be μ-Hölder continuous, $\Phi \in C^{\mu}(U)$ if

$$\sup_{x,y\in U, x\neq y} \frac{|\Phi(x) - \Phi(y)|}{|x - y|^{\mu}} < \infty.$$

Similarly, one defines the class $C^{1,\mu}(U)$ to consist of all functions $\Phi \in C^1(U)$ such that $\nabla\Phi \in C^{\mu}(U)$. The above two definitions extends in a natural way to the case of vector-valued mappings.

We say that a domain $\Omega \subset \mathbb{R}^n$ has $C^{1,\alpha}$ boundary if there is a $C^{1,\alpha}$ diffeomorphism $G : \overline{B} \to \overline{\Omega}$.

Pavlović in [26] showed that harmonic quasiconformal mappings of the unit disk in $\mathbb{R}^2$ onto itself are bi-Lipschitz mappings. From then, several important results have been obtained regarding harmonic quasiconformal mappings in $\mathbb{R}^2$ and the Lipschitz continuity. The second author in [8] proved that every quasiconformal harmonic mapping between Jordan domains with $C^{1,\alpha}$ boundaries is Lipschitz continuous on the closure of domain. The result in [8] was extended in [9] for Jordan domains with only Dini's smooth boundaries. Lately, in [13] it was proved the Hölder continuity (but in general, Lipschitz continuity does not hold) of a harmonic quasiconformal mapping between two Jordan domains having only C^1 boundaries. Other important results for $n = 2$ with different conditions and settings can be found in [1, 4, 6, 11, 12, 15, 16, 18–20, 23, 24] and in their references.

For higher dimensional case there are some important results also (see e.g. [2, 10, 17, 21]). In [10] it was proven that a quasiconformal mapping of the unit ball onto a domain with C^2 smooth boundary, satisfying Poisson differential inequality, is Lipschitz continuous. This implies that harmonic quasiconformal mappings from unit ball B to Ω with C^2 boundary are Lipschitz continuous. This was also proved by Astala and Manojlovic in [2] using a slight modification of the following statement also proved there: a harmonic K-quasiconformal mapping from B to B is Lipschitz with the Lipschitz constant depending on the value of K, dimension of n and $\operatorname{dist}(f(0), S)$.

Our main result generalizes the result in [8] and improves the mentioned corollaries in [2] and [10]. It reads as follow.

Theorem 1.1 *Let $f : B \to \mathbb{R}^n$ be a quasiconformal harmonic (qch) mapping, $f(B) = \Omega$, and $\partial\Omega \in C^{1,\alpha}$. Then f is Lipschitz continuous in B.*

The proof of the corresponding result for 2-dimensional case in [8] uses conformal mappings, however conformal mappings in higher-dimensional setting are very rigid, and this is why we need to find another way to deal with the proof of Theorem 1.1. The initial idea lies on the following simple approach. Let $\eta \in S$ and $f(\eta) = q \in \partial\Omega$. We can suppose that $q = 0$ and the tangent plane of q at $\partial\Omega$ is $x_n = 0$. This can be obtained in the following way: Using a isometry L we can postcompose f such that we get a function $\tilde{f}$ from B to Ω', $\tilde{f}(\eta) = 0$ and the tangent plane of this point on $\partial\Omega'$ is $x_n = 0$. Observe that $\tilde{f}$ is also harmonic and quasiconformal, because it is composed by a isometry. The Lipschitz continuity for function $\tilde{f}$ would yield the proof of this property for the function f also, because the isometry preserves the distances.

The proof is given in Section 3. It uses an iteration procedure. Before that, in next section, we give some basic preparations through Theorems 2.1-2.4.

2 Auxiliary Results

The next theorem is of general interest; on the other side it plays an important role in proving Theorem 1.1. Some versions of it for $n = 2$ can be found in [7] and [22].

Theorem 2.1 *Let $u : \overline{B} \subset \mathbb{R}^n \to \mathbb{R}$, $n \geq 3$, be a real harmonic function, $\eta \in S$. Assume that $|u(\xi) - u(\eta)| \leq M|\xi - \eta|^{\mu}$, $\forall \xi \in S$, for some $\mu \in (0, 1)$. Then we have $C = C(M, \mu, n)$ such that*

$$|\nabla u(x)|(1 - |x|)^{1-\mu} \leq C,$$

where $x = r\eta$, $r \in [0, 1)$.

Proof Throught the proof, the constant C can change its value. Using the Poisson integral formula we have

$$u(x) = \int_S \frac{1 - |x|^2}{\left(1 + |x|^2 - 2(\xi, x)\right)^{\frac{n}{2}}} u(\xi) d\sigma(\xi).$$

Observe that

$$\nabla u(x) = \int_S Q(x, \xi) u(\xi) d\sigma(\xi), \tag{2.1}$$

where

$$\begin{aligned} Q(x,\xi) &= \frac{(-2x)\left(1 + |x|^2 - 2(\xi, x)\right)^{\frac{n}{2}} - n(1 - |x|^2)\left(1 + |x|^2 - 2(\xi, x)\right)^{\frac{n}{2}-1}(x - \xi)}{\left(1 + |x|^2 - 2(\xi, x)\right)^{n}} \\ &= \frac{(-2x)\left(1 + |x|^2 - 2(\xi, x)\right) - n(1 - |x|^2)(x - \xi)}{\left(1 + |x|^2 - 2(\xi, x)\right)^{\frac{n}{2}+1}} \\ &= \frac{(-2x)\left(1 + |x|^2 - 2(\xi, x)\right) - n(1 - |x|^2)(x - \xi)}{\left(1 + |x|^2 - 2(\xi, x)\right)} \cdot \frac{1}{\left(1 + |x|^2 - 2(\xi, x)\right)^{\frac{n}{2}}}. \end{aligned} \tag{2.2}$$

Let $h \in R^n$ be an arbitrary vector. Then

$$\langle \nabla u(x), h\rangle = \int_S \langle Q(x,\xi), h\rangle u(\xi) d\sigma(\xi). \tag{2.3}$$

Since (2.3) is true for every harmonic function $u : \overline{B} \to \mathbb{R}$, taking the constant function $u(\eta)$, we get

$$0 = \int_S \langle Q(x,\xi), h\rangle u(\eta) d\sigma(\xi), \tag{2.4}$$

which, together with (2.3), gives us

$$\langle \nabla u(x), h\rangle = \int_S \langle Q(x,\xi), h\rangle [u(\xi) - u(\eta)] d\sigma(\xi). \tag{2.5}$$

On the other side

$$\begin{aligned} &\left|\frac{-2\langle x,h\rangle\left(1+|x|^2-2\langle\xi,x\rangle\right) - n(1-|x|^2)\langle x-\xi,h\rangle}{\left(1+|x|^2-2\langle\xi,x\rangle\right)}\right| \\ &\le 2|x||h| + n\frac{\left(1-|x|^2\right)|x-\xi||h|}{|x-\xi|^2} \le \\ &= 2|x||h| + 2n|h|\frac{1-|x|}{|x-\xi|} \le (2+2n)|h|. \end{aligned} \tag{2.6}$$

In the last inequality it is used the fact that $1-|x| \le |x-\xi|$, which is obviously true from the geometrical point of view, but it is also equivalent to $\langle\xi, x\rangle \le |x|$ (Cauchy-Schwarz inequality).

From (2.2), (2.5), (2.6) we get

$$|\langle \nabla u(x), h\rangle| \le (2n+2)|h| \int_S \frac{|u(\xi)-u(\eta)|}{\left(1+|x|^2-2\langle\xi,x\rangle\right)^{\frac{n}{2}}} d\sigma(\xi) \tag{2.7}$$

As h was taken arbitrary, then

$$|\nabla u(x)| \le (2n+2) \int_S \frac{|u(\xi)-u(\eta)|}{\left(1+|x|^2-2\langle\xi,x\rangle\right)^{\frac{n}{2}}} d\sigma(\xi), \tag{2.8}$$

which is equivalent to

$$\begin{aligned} |\nabla u(r\eta)| &\le (2n+2) \int_S \frac{|u(\xi)-u(\eta)|}{\left(1+r^2-2r\langle\xi,\eta\rangle\right)^{\frac{n}{2}}} d\sigma(\xi) \\ &= (2n+2) \int_S \frac{|u(\xi)-u(\eta)|}{\left((1-r)^2+r|\xi-\eta|^2\right)^{\frac{n}{2}}} d\sigma(\xi), \end{aligned} \tag{2.9}$$

where $x = r\eta$, $r = |x| \in [0,1)$.

Using the condition of the theorem we get

$$|\nabla u(r\eta)| \le M(2n+2) \int_S \frac{|\xi-\eta|^\mu}{\left((1-r)^2+r|\xi-\eta|^2\right)^{\frac{n}{2}}} d\sigma(\xi). \tag{2.10}$$

Because of the symmetry, it is enough to show the required inequality for $\eta = (1, 0, \ldots, 0)$.

1st Case $r = |x| \geq \frac{1}{2}$.

As the integrand function in (2.10) depends only on the first coordinate of ξ, we use the following representation ([3], Appendix A5):

$$|\nabla u(r\eta)| \leq M(2n+2)C_1 \int_{-1}^{1} \int_{S_{n-2}} \frac{(2-2x)^{\frac{\mu}{2}}}{\left((1-r)^2 + r(2-2x)\right)^{\frac{n}{2}}} \left(1-x^2\right)^{\frac{n-3}{2}} d\sigma_{n-2}(\zeta) dx,$$

where σ_{n-2} denotes the respective normalized surface-area measure on the unit sphere S_{n-2} in $\mathbb{R}^{n-1}$. The constant C_1 depends on n and the volumes of the unit balls in $\mathbb{R}^n$ and $\mathbb{R}^{n-1}$. From this, it follows

$$\begin{aligned} |\nabla u(r\eta)| &\leq C \int_{S_{n-2}} d\sigma_{n-2}(\zeta) \int_{-1}^{1} \frac{(2-2x)^{\frac{\mu}{2}}}{\left((1-r)^2 + r(2-2x)\right)^{\frac{n}{2}}} \left(1-x^2\right)^{\frac{n-3}{2}} dx \qquad (2.11) \\ &\leq C \int_{-1}^{1} \frac{(2-2x)^{\frac{\mu}{2}}}{\left((1-r)^2 + r(2-2x)\right)} \frac{2^{\frac{n-3}{2}}(1-x)^{\frac{n-3}{2}}}{\left((1-r)^2 + r(2-2x)\right)^{\frac{n-2}{2}}} dx \\ &= C 2^{\frac{\mu}{2}} 2^{\frac{n-3}{2}} \int_{-1}^{1} \frac{(1-x)^{\frac{\mu-1}{2}}}{\left((1-r)^2 + r(2-2x)\right)} \left(\frac{1-x}{(1-r)^2 + r(2-2x)}\right)^{\frac{n-2}{2}} dx. \end{aligned}$$

Since $r \geq \frac{1}{2}$, we easily get

$$\frac{1-x}{(1-r)^2 + r(2-2x)} \leq \frac{1-x}{(1-r)^2 + (1-x)} \leq 1,$$

so

$$|\nabla u(r\eta)| \leq C \int_{-1}^{1} \frac{(1-x)^{\frac{\mu-1}{2}}}{(1-r)^2 + (1-x)} dx. \qquad (2.12)$$

First, using the substitution $1 - x = t^2$, then $s = \frac{t}{1-r}$, we have

$$|\nabla u(r\eta)| \leq C \int_{0}^{\sqrt{2}} \frac{2t^{\mu}}{(1-r)^2 + t^2} dt = C \frac{(1-r)^{\mu}}{(1-r)^2} \int_{0}^{\frac{\sqrt{2}}{1-r}} \frac{2s^{\mu}}{1+s^2} (1-r) ds,$$

so

$$|\nabla u(r\eta)| \leq C(1-r)^{\mu-1} \int_{0}^{\infty} \frac{s^{\mu}}{1+s^2} ds.$$

As the last integral converges we finally have

$$|\nabla u(r\eta)|(1-r)^{1-\mu} \leq C, \qquad (2.13)$$

for $r \in [\frac{1}{2}, 1)$, where C depends on M, μ and n only.

2nd Case $r = |x| < \frac{1}{2}$

Here the proof is quite straightforward. Since

$$\frac{|\xi - \eta|^{\mu}(1-r)^{1-\mu}}{((1-r)^2 + r|\xi - \eta|^2)^{\frac{n}{2}}} < \frac{2^{\mu} \cdot 1}{\left(\frac{1}{2}\right)^{n}} = 2^{n+\mu}, \qquad (2.14)$$

using (2.10) we get

$$|\nabla u(r\eta)|(1-r)^{1-\mu} \leq M(2n+2)2^{n+\mu}. \qquad (2.15)$$

We conclude that the inequality is true for all $r \in [0, 1)$, with the final C being the larger of the obtained constants on the RHS of (2.13) and (2.15). □

The idea of the proof in Section 3 will be based on obtaining locally the C^μ condition of f on the unit sphere for $\mu < 1$, by increasing μ. In relation to a fixed point $\eta \in S$ this will, in one moment, give us a similar inequality as the one from Theorem 2.1, but for $\mu > 1$. So, on this step, we need a different version of the previuos statement which is given in the following theorem. However, the proof of it is very similar to the proof of the previous one.

Theorem 2.2 *Let $u : \overline{B} \subset \mathbb{R}^n \to \mathbb{R}$, be a harmonic function, $\eta \in S$. Assume that $|u(\xi) - u(\eta)| \le M|\xi - \eta|^\mu$, $\forall \xi \in S$, for some $\mu > 1$. Then we have $C = C(M, \mu, n)$ such that*

$$|\nabla u(r\eta)| \le C,$$

for every $r \in [0, 1)$.

Proof The proof of the theorem for $r \in [\frac{1}{2}, 1)$ is identical to the previous theorem until (2.12).

$$\int_{-1}^{1} \frac{(1-x)^{\frac{\mu-1}{2}}}{(1-r)^2 + (1-x)} dx \le \int_{-1}^{1} (1-x)^{\frac{\mu-3}{2}} dx = \frac{2^{\frac{\mu+1}{2}}}{\mu - 1}$$

shows that the inequality is true.

For $r \in [0, \frac{1}{2})$, similar to (2.14) we see that

$$\frac{|\xi - \eta|^\mu}{((1-r)^2 + r|\xi - \eta|^2)^{\frac{n}{2}}}$$

is bounded, so therefore again from (2.10) we have our inequality. □

The next celebrated theorem will also be used. The proof can be found in [5].

Theorem 2.3 (Mori's theorem) *Let g be a K-quasiconformal mapping of B onto B, $n \ge 2$, with $g(0) = 0$. Then*

$$|g(x) - g(y)| \le M(n, K)|x - y|^\beta,$$

for all $x, y \in B$, where $\beta = K^{\frac{1}{1-n}}$.

We collect now the following useful result. The proof can be found in [25]. We will formulate it in the form which corresponds to our notation and use.

Theorem 2.4 *Let u be a real harmonic function on $\overline{B}$ and $\mu \in (0, 1)$ such that*

$$||u(r\eta)| - |u(\eta)|| \le C(1-r)^\mu, \forall r \in [0, 1), \eta \in S, \tag{2.16}$$

where C is independent of r and η, then u is μ-Hölder continuous in $\overline{B}$, i.e.:

$$|u(x) - u(y)| \le M|x - y|^\mu,$$

for all $x, y \in \overline{B}$.

Using the previous theorem we can easily prove the following lemma.

Lemma 2.5 *Let u be a real harmonic function on $\overline{B}$ and $\mu \in (0, 1)$ such that*

$$|\nabla u(r\eta)| \le C(1-r)^{\mu-1}, \forall r \in (0, 1), \eta \in S,$$

where C does not depend on r and η, then u is μ-Hölder continuous in $\overline{B}$.

Proof To prove this lemma, from Theorem 2.4, it is sufficient to show the relation (2.16).

We have

$$u(\eta) - u(r\eta) = \int_{\gamma_r} D_1 u dx_1 + \ldots + D_n u dx_n, \tag{2.17}$$

where γ_r is the radial segment with endpoints $r\eta$ and η.

Therefore, we have

$$||u(r\eta)| - |u(\eta)|| \le |u(r\eta) - u(\eta)| \le \int_r^1 |\langle \nabla u(t\eta), \eta\rangle| dt$$

$$\le C \int_r^1 (1-t)^{\mu-1} dt \le C\frac{(1-r)^{\mu}}{\mu}. \tag{2.18}$$

□

3 Proof of the Main Result - Theorem 1.1

Proof First, let we prove the Hölder continuity of $f = (f_1, \ldots, f_n)$. Indeed, let G be a quasiconformal diffeomorphism (recall that Ω has $C^{1,\alpha}$ boundary) from B^n to Ω which is Lipschitz continuous mapping up to the boundary, such that $G(0) = f(0)$. Then the mapping $g = G^{-1} \circ f$ is a K' quasiconformal mapping (as a composition of two quasiconformal mappings) of B onto B, where $g(0) = 0$. According to Mori's Theorem 2.3, there exists a constant $M_1(n, K')$ such that

$$|g(x) - g(y)| \le M_1(n, K')|x-y|^{K'^{\frac{1}{1-n}}},$$

for all $x, y \in B^n$.

As $f = G \circ g$, then f satisfies a similar inequality, being a composition of Lipschitz and Hölder continuous functions:

$$|f(x) - f(y)| \le C_1|x-y|^{\beta}, \tag{3.1}$$

for all $x, y \in \overline{B^n}$, where $\beta \in (0, 1)$, and the constant C_1 depends on M_1 and the Lipschitz constant of G.

In view of the remark after the formulation of Theorem 1.1, there exists a neighbourhood $\mathcal{O}$ of the origin in R^{n-1} which is the projection of $\partial\Omega \cap B(0,\rho)$ in R^{n-1} and a $C^{1,\alpha}$ function $\Phi : \mathcal{O} \to \mathbb{R}$ such that $\partial\Omega \cap B(0,\rho)$ can be expressed as the graph of Φ, i.e. points of $\partial\Omega \cap B(0,\rho)$ are of the form:

$$(\zeta_1, \ldots, \zeta_{n-1}, \Phi(\zeta_1, \ldots, \zeta_{n-1})), \tag{3.2}$$

where $(\zeta_1, \ldots, \zeta_{n-1}) \in \mathcal{O}$.

The function Φ has the properties $\Phi(0, \ldots, 0) = 0$ and $D_j\Phi(0, \ldots, 0) = 0$, for all $j \in \{1, 2, \ldots, n-1\}$, and

$$|\nabla\Phi(\zeta) - \nabla\Phi(\omega)| \le C_2|\zeta - \omega|^{\alpha}. \tag{3.3}$$

The constant C_2 is the same for all points $q \in \partial\Omega$, because of the $C^{1,\alpha}$ condition of $\partial\Omega$. Also,

$$|\Phi(\zeta) - \Phi(\omega)| = |\langle\nabla\Phi(c), \zeta - \omega\rangle| \le |\nabla\Phi(c)||\zeta - \omega|, \tag{3.4}$$

where c belongs to the segment $[\zeta, \omega]$.

Using (3.3) we get

$$
\begin{aligned}
|\nabla\Phi(c)| &\le |\nabla\Phi(\zeta)| + |\nabla\Phi(c) - \nabla\Phi(\zeta)| &(3.5)\\
&\le C_2(|\zeta|^\alpha + |c-\zeta|^\alpha) \le C_2\left(|\zeta|^\alpha + |\zeta-\omega|^\alpha\right),\\
|\nabla\Phi(c)| &\le |\nabla\Phi(\omega)| + |\nabla\Phi(c) - \nabla\Phi(\omega)| &(3.6)\\
&\le C_2(|\omega|^\alpha + |c-\omega|^\alpha) \le C_2\left(|\omega|^\alpha + |\zeta-\omega|^\alpha\right),
\end{aligned}
$$

which yields

$$|\nabla\Phi(c)| \le C_2 \min\left\{\{|\zeta|^\alpha, |\omega|^\alpha\} + |\zeta-\omega|^\alpha\right\}.$$

Therefore, from (3.4) we have:

$$|\Phi(\zeta) - \Phi(\omega)| \le C_2|\zeta-\omega|\left(\min\left\{|\zeta|^\alpha, |\omega|^\alpha\right\} + |\zeta-\omega|^\alpha\right), \tag{3.7}$$

for all ζ, ω in $\mathcal{O}$.

Let $F = (F_1, \dots, F_n) = f|_S$ or $P[F] = f$. Notice that F is also C^β in S. We will use the notation $\tilde{F}(\xi) = (F_1(\xi), \dots, F_{n-1}(\xi))$. $\tilde{F}$, as F, also satisfies (3.1). In view of (3.2) we have that in a small neighbourhood of η in S, F_n is of the form

$$F_n(\xi) = \Phi(F_1(\xi), \dots, F_{n-1}(\xi)).$$

We may also assume that this neighbourhood of η is of the form $V(\eta) = B(\eta, \delta) \cap S$, where δ is small enough positive constant for all $q \in \partial\Omega$. Indeed, let $\tilde{U}(q) = B(q, r_q) \cap \partial\Omega$ be the neighbourhood of q in $\partial\Omega$ such that after the isometry L_q (the one that sends q to 0 and which makes the plane $x_n = 0$ the tangent plane of $\partial\Omega$ at point 0), $L_q(\tilde{U}(q))$ is the neighbourhood of 0 which is the graphic of a function as in (3.2). Furthermore, we can choose r_q small enough, such that for every point $p \in \tilde{U}(q)$, the image of $\tilde{U}(q)$ under the respective isometry L_p is a graphic of a function.

Observe now $U(q) = B\left(q, \frac{r_q}{2}\right) \cap \partial\Omega$. The collection $\{U(q)\}_{q\in\partial\Omega}$ is a cover of $\partial\Omega$. As $\partial\Omega$ is compact, there exists a finite subcollection $\{U(q_k)\}_{k=1}^m$ which covers $\partial\Omega$. Let $\rho = \min\left\{\frac{r_{q_1}}{2}, \dots, \frac{r_{q_m}}{2}\right\}$. Since F is continuous on a compact, there is a $\delta > 0$ such that if $|\xi_1 - \xi_2| < \delta$, $\xi_1, \xi_2 \in S$, then $|F(\xi_1) - F(\xi_2)| < \frac{\rho}{2}$. This ensures that the image of every $V(\eta) = B(\eta, \delta) \cap S$ under F is contained in a $B(q_j, r_{q_j}) \cap \partial\Omega = \tilde{U}(q_j)$, and further, after the mentioned isometry is done, this image is the graphic of a function as in (3.2).

We get back to our fixed η, such that $f(\eta) = 0$. Now

$$
\begin{aligned}
|F_n(\xi) - F_n(\eta)| &= \left|\Phi\left(\tilde{F}(\xi)\right) - \Phi(0)\right| &(3.8)\\
&\le C_2|\tilde{F}(\xi)|\left(\min\left\{|\tilde{F}(\xi)|^\alpha, 0\right\} + |\tilde{F}(\xi) - 0|^\alpha\right)\\
&= C_2|\tilde{F}(\xi)|^{1+\alpha} \le C_1^{1+\alpha} C_2 |\xi - \eta|^{(1+\alpha)\beta},
\end{aligned}
$$

for all $\xi \in V(\eta)$. The function F_n is bounded, because $F = f|_S$ is bounded ($|F(\xi)| \le \widetilde{M}$, for all $\xi \in S$), so if $\xi \in S\setminus V(\eta)$ then

$$|F_n(\xi) - F_n(\eta)| \le 2\widetilde{M} \le \frac{2\widetilde{M}}{\delta^{(1+\alpha)\beta}}|\xi - \eta|^{(1+\alpha)\beta}. \tag{3.9}$$

Taking $M = \max\left\{C_1^{1+\alpha} C_2, \frac{2\widetilde{M}}{\delta^{(1+\alpha)\beta}}\right\}$ we get

$$|F_n(\xi) - F_n(\eta)| \le M|\xi - \eta|^{(1+\alpha)\beta}, \tag{3.10}$$

for all $\xi \in S$.

Now, from Theorem 2.1, we have

$$|\nabla f_n(r\eta)| \le C(1-r)^{(1+\alpha)\beta-1}, \forall r \in [0,1).$$

As f is quasiconformal mapping then

$$\frac{\max_{|h_1|=1} |f'(x)h_1|}{\min_{|h_2|=1} |f'(x)h_2|} \le K < \infty, \forall x \in B.$$

Taking, $h_1 = e_j$ and $h_2 = e_n$, for $x = r\eta$ we have

$$|\nabla f_j(r\eta)| \le K|\nabla f_n(r\eta)| \le K \cdot C(1-r)^{(1+\alpha)\beta-1},$$

for all $j \in \{1, \dots, n-1\}$.

This implies

$$|\nabla f_j(r\eta)| \le C(1-r)^{(1+\alpha)\beta-1}, \tag{3.11}$$

where C is a new global constant for all $j \in \{1, \dots, n\}$, and all $r \in [0,1)$.

We want to prove (3.11) in B. Let $\eta_1 \ne \eta$ be an arbitrary point on S and $f(\eta_1) = q_1$. Let L_{q_1} be the isometry that sends q_1 to 0, with $x_n = 0$ being the tangent plane of $L_{q_1}(\partial\Omega)$ at $L_{q_1}(q_1) = 0$.

Let $L_{q_1} \circ f = \tilde{f} = \left(\tilde{f}_1, \dots, \tilde{f}_n\right)$. Then $\tilde{f}$ has all the properties of the function f with η_1 in place of η: at $\tilde{f}(\eta_1) = 0$ the tangent plane of the surface $L_{q_1}(\partial\Omega)$ is $x_n = 0$ and $\tilde{f}(\eta_1)$ has a neighbourhood in $L_{q_1}(\partial\Omega)$ which can be expressed as a part of a graphic of the form (3.2). Using the same procedure, we conclude that

$$|\nabla \tilde{f}_j(r\eta_1)| \le C(1-r)^{(1+\alpha)\beta-1},$$

for all $j \in \{1, \dots, n\}$, and all $r \in [0,1)$. Constant C is universal and it does not depend on η_1, because δ and M are independent of the choice of $\eta \in S$. As $f = L_{q_1}^{-1}\tilde{f}$, $\left(L_{q_1}^{-1}\text{ is also an isometry}\right)$ we get

$$f_j(\xi) = b_j + \sum_{k=1}^{n} a_{j,k}\tilde{f}_k(\xi),$$

$j \in \{1, \dots, n\}$, so

$$\nabla f_j(\xi) = \sum_{k=1}^{n} a_{j,k}\nabla \tilde{f}_k(\xi) \tag{3.12}$$

where $\{a_{i,j}\}_{1\le i,j\le n}$ is an orthogonal matrix. From (3.12) we have:

$$\begin{aligned} |\nabla f_j(\xi)| &\le \sum_{k=1}^{n} |a_{j,k}||\nabla \tilde{f}_k(\xi)| \qquad (3.13)\\ &\le \left(\sum_{k=1}^{n} |\nabla \tilde{f}_k(\xi)|^2\right)^{\frac{1}{2}}. \end{aligned}$$

In the last inequality it is used the Cauchy-Schwarz inequality and the orthogonality of matrix $\{a_{j,k}\}_{1\le j,k\le n}$. Taking $\xi = r\eta_1$ we get

$$|\nabla f_j(r\eta_1)| \le \sqrt{n}C(1-r)^{(1+\alpha)\beta-1}.$$

As the point η_1 was arbitrary we conclude

$$|\nabla f_j(x)| \le C(1-r)^{(1+\alpha)\beta-1}, r = |x|,$$

for all $x \in B$.

From Lemma 2.5 it follows that $f_j \in C^{(1+\alpha)\beta}(\overline{B})$, for all $j \in \{1, \dots, n\}$ and so $f \in C^{(1+\alpha)\beta}(\overline{B})$.

We could have chosen $\beta < \frac{1}{2}$ (by decreasing it, if necessary) so the numbers $(1+\alpha)^k\beta \ne 1$, for every k. As $1+\alpha > 1$ there exists a unique integer k_0 such that $(1+\alpha)^{k_0}\beta < 1$ and $(1+\alpha)^{k_0+1}\beta > 1$. Repeating the procedure, we get that $f \in C^{(1+\alpha)^2\beta}(\overline{B}), \dots, C^{(1+\alpha)^{k_0}\beta}(\overline{B})$. Note that such procedure for two-dimensional setting and different purpose has been used in [22] and in [14]. Similar to (3.8) it follows that $|F_n(\xi) - F_n(\eta)| \le M|\xi - \eta|^{(1+\alpha)^{k_0+1}\beta}, \forall\xi \in S$. This time, using Theorem 2.2 we obtain

$$|\nabla f_n(r\eta)| \le C, \forall r \in [0, 1).$$

Using the same order of implications, first we get the same inequality for every f_k on points $r\eta$. Then, using the isometries, we get the inequality on every point of B for a global constant C. This implies trivially, by mean value inequality, the Lipschitz continuity of function f in $\overline{B}$. □

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ODLUKU
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3. **Darko Mitrović** je rukovodilac bilateralnog projekta "Problemi toka na mnogostrukostima" finansiranog od strane Ministarstva nauke Crne Gore i Ministarstva nauke Austrije u periodu 2015-2017.
4. **Darko Mitrović** je bio rukovodilac hrvatsko-crnogorskog bilateralnog projektas "Transport u izrazito heterogenim sredinama" finansiranog od strane Ministarstva znanosti, obrazovanja i športa republike Hrvatske i Ministarstva nauke Crne Gore u periodu 2012-2014.

Predavanja po pozivu

1. Mitrovic, D.: **Singular solutions for systems of conservation laws**, Entropy and singular solutions to conservation laws: Pressureless Gas dynamics and other application, Morgantown, USA, 26.-28.09.2014. (http://math.wvu.edu/entropy2014/)

2. Mitrovic, D.: **H-distributions and applications on velocity averaging for transport equations**, Contemporary Topics in Conservation Laws, Besancon, France on February 9-12, 2015.

Bibliografija

E-SCI i SCI časopisi

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26. *Mišur, M; Mitrovic, D.:* **On a generalization of compensated compactness in the L^p-L^q setting**, *Journal of Functional Analysis,* 268 (2015) 1904-1927;

25. *Andreianov, B; Mitrovic, D.:* **Entropy conditions for scalar conservation laws with discontinuous flux revisited**, *Annales de l'Institut Henri Poincaré (C) Analyse Non Linéaire,* doi:10.1016/j.anihpc.2014.08.002

24. *Mitrovic, D.; Nordbotten, J.M.; Kalisch, H.:* **Dynamics of the interface between immiscible liquids of different densities with low Froude number**, *Nonlinear Analysis Real World Applications,* 15 (2014), 361–366

23. *Aleksic, J.; Mitrovic, D.:* **Strong traces for averaged solutions of heterogeneous ultra-parabolic transport equations**, *J. of Hyperbolic Differential Equations 4 (2013), 659-676.*

22. *Lazar, M; Mitrovic, D.:* **On an extension of a bilinear functional on $L^p(R^d)\otimes E$ to a Bochner space with an application on velocity averaging**, *C. R. Acad. Sci. Paris Ser. I Math. 351 (2013), 261--264.*

21. *Mitrovic, D.:* **On a Leibnitz type formula for fractional derivatives**, *Filomat 27:6 (2013), 1141–1146.*

20. *Kalisch, H.; Mitrovic, D.:* **Singular solutions for the shallow water equations**, *IMA J. Appl. Maths. 77 (2012), 340-350.*

19. *Kalisch, H.; Mitrovic, D.:* **Singular solutions of a fully nonlinear 2x2 system of conservation laws**, *Proceedings of the Edinburgh Mathematical Society, 55 (2012), 711-729.*

18. *Lazar, M.; Mitrovic, D.:* **Velocity averaging – general framework**, *Dynamics of Partial Differential Equations, Vol.9, No.3, 239-260, 2012.*

17. *Antonic, N.; Mitrovic, D.:* **H-distributions—an extension of the H-distributions in the L^p-L^q setting**, *Volume 2011 (2011), Article ID 901084, 12 pages, doi:10.1155/2011/901084*

16. *Mitrovic, D.; Ivec, I.:* **A Generalization of H-measures and Application on Purely Fractional Scalar Conservation Laws**, *Communication on Pure and Applied Analysis, Volume 10, Number 6, November 2011, 1617 - 1627.*

14. *Lazar, M.; Mitrovic, D.:* **The velocity averaging for a heterogeneous heat type equation**, *Mathematical Communications, 16(2011), 271-282.*

13. *Danilov, V.G.; Mitrovic, D.:* **Shock Wave Formation Process for a Multidimensional Scalar Conservation Law**, *Quarterly of Applied Mathematics, 69 (2011), 613-634.*

12. *Mitrovic, D.:* **New Entropy Conditions for Scalar Conservation Laws with Discontinuous Flux**, *Discrete and Continuous Dynamical Systems-A, Vol. 30, August 2011 (20 pages, 4 figures)*

11. *Mitrovic, D.:* **Existence and Stability of Multidimensional Scalar Conservation Laws with Discontinuous Flux**, *Networks and Heterogeneous Media, Vol.5 (2010), 163-188*

10. *Mitrovic, D.; Bojkovic, V.; Danilov, V.G.:* **Linearization of the Riemann problem for a triangular system of conservation laws and delta shock wave formation process**, *Mathematical Methods in the Applied Sciences, Vol. 33 (2010), 904 - 921*

9. *Holden, H.; Karlsen, K.H.; Mitrovic, D.; Panov, E.Yu.:* **Strong compactness of approximate solutions to degenerate elliptic-hyperbolic equations with discontinuous flux functions**, *Acta Mathematica Scientia B (issue dedicated to J.GLimm 75th birthday), Vol. 29 (2009), 1573-1672*

8. *Aleksic, J.; Mitrovic, D.:* **On the compactness for two dimensional scalar conservation laws with discontinuous flux**, *Communications in Mathematical Sciences, Vol. 7 (2009), 963-971.*

7. *Aleksic, J.; Mitrovic, D.; Pilipovic, S.:* **Hyperbolic conservation laws with vanishing nonlinear diffusion and linear dispersion in heterogeneous media**, *Journal of Evolution Equations, Vol. 9 (2009), 809-828.*

6. *Mitrovic, D.:* **On the heat equation involving the δ-distribution as a coefficient**, *Mathematical and Computer Modeling, 50 (2009) 109-115*

5. *Danilov, V. G.; Mitrovic, D.:* **Smooth Approximations of Global in Time Solutions to Scalar Conservation Laws**, *Abstract and Applied Analysis, Volume 2009, Article ID 350762, 26 pages*

4. *Danilov, V. G.; Mitrovic, D.:* **Delta shock wave formation in the case of triangular hyperbolic system of conservation laws**, *Journal of Differential Equations, 245(2008) 3704-3734*

3. *Mitrovic, D.; Nedeljkov, M.:* **Delta shock waves as a limit of shock waves**, *Journal of Hyperbolic Differential Equations, Vol 4., No. 4 (2007), 629-653*

2. *Mitrovic, D.; Pilipovic, S.:* **Approximations of linear Dirichlet problems with singularities**, *J. Math. Anal. Appl. 313 (2006), No. 1, 98-119.*

1. *Danilov, V.; Mitrovic, D.:* **Weak asymptotics of shock wave formation process**, *Nonlinear Anal. 61 (2005), No. 4, 613-635.*

Ostale publikacije

1. *Mitrovic, D.:* **Scalar conservation law with discontinuous flux - thickened entropy conditions and doubling of variables**, *Mathematica Aeterna, Vol. 1, 2011, no. 03, 163 --172*

2. *Holden, H.; Karlsen, K.H.; Mitrović, D:* **Zero diffusion dispersion limits for scalar conservation law with discontinuous flux function**, *International Journal of Differential Equations, Volume 2009,* Article ID 279818, *33 pages.*

3. *Bojkovic, V.; Mitrovic, D.:* **A characterization of Riemann invariants for 2×2 system of hyperbolic conservation laws,** *Journal of Mathematical Sciences: Advances and Applications, Vol. 1, Number 3 (2008), 579-586*

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5. *Mitrovic, D.:* **Singularity formation for a pressureless gas dynamics system of conservation laws,** *IEEE Catalog No. 06EX1351, ISBN 5-9651-0226-7, Days on Diffraction 2006, 197-208,* (http://ieeexplore.ieee.org/xpl/freeabs_all.jsp?arnumber=4154034)

6. *Danilov, V. G.; Mitrovic, D.:* **New approach to shocks generation for conservation laws. Example: global solution to Hopf equation,** *Matematički Vesnik, 56 (2004), No. 1-2,* 23-46.

7. *Mitrovic, D.:* **Uniform in teR description of shock wave formation process and application to convex scalar conservation law,** *Mathematica Montisnigri, Vol XVII (2004) 37-55.*

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Dr DAVID KALAJ bira se u akademsko zvanje redovni profesor Univerziteta Crne Gore za predmete: Kompleksna analiza 2 (studijski program Matematika), Analiza 3 (studijski program Računarske nauke) i Analiza 3 (studijski program Fizika) na Prirodno-matematičkom fakultetu.

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REKTOR
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Biografija dr Davida Kalaj

Lični podaci

Ime i prezime: David Kalaj

Državljanstvo: Crna Gora

Datum i mjesto rođenja: 11. 12. 1971. Podgorica

Institucija

Univerzitet Crne Gore,

- Prirodno-matematički fakultet, UCG

Zvanje: Redovni profesor

- Samostalni studijski program za obrazovanje učitelja na albanskom jeziku, UCG

Funkcija: rukovodilac

A) OBRAZOVANJE

1991: Maturirao Srednju školu „25 Maj", Tuzi, Titograd, sa odličnim uspjehom,

1995: Diplomirao na Prirodno-Matematičkom Fakultetu, Univerziteta Crne Gore sa prosječnom ocjenom 9.52. (Za postignuti uspjeh tokom školovanja je 1995. godine dobio Studensku nagradu 19 decembar (Nagrada opštine Podgorica))

1995: Magistrirao na Matematičkom fakultetu, Univerziteta u Beogradu, smjer matematička analiza, sa temom *Harmonijske funkcije i kvazikonformna preslikavanja*, pošto je položio sve predviđene ispite sa ocjenom 10.

2002: Odbranio doktorsku disertaciju pod nazivom *Harmonijske funkcije i kvazikonformne harmonijske funkcije između konveksnih domena* na Matematičkom fakultetu, Univerziteta u Beogradu.

B) NASTAVNA ISKUSTVA

1995 – 1997 | Saradnik u nastavi, Prirodno-Matematičkog fakulteta UCG

1998 – 2002 | Asistent, Prirodno-Matematičkog fakulteta UCG

2002 – 2007 | Docent Prirodno-Matematičkog fakulteta UCG

Kompleksna Analiza, Matematička Analiza 3 (PMF), Matematike 1,2,3,4 (Studijski programi za obrazovanje učitelja na albanskom jeziku)

2007- 2012 | Vanredni professor Prirodno-Matematičkog fakulteta UCG

Kompleksna Analiza, Mathematička Analiza 3 (PMF),

Matematike 1,2,3,4 (Studijski program za obrazovanje učitelja na albanskom jeziku)

Realna i kompleksna analiza (kurs na posdiplomskim studijama PMF-a)

Viša analiza (kurs na doktorskim studijama PMF i Matematičkog fakulteta u Beogradu)

2012 - | Redovni profesor Prirodno-Matematičkog fakulteta UCG

- Kompleksna Analiza II, Mathematička Analiza 3, Mathematička ... (PMF),
- Matematike 1,2,3,4 (Studijski program za obrazovanje učitelja na albanskom jeziku)

- Realna i kompleksna analiza (kurs na posdiplomskim studijama [illegible])
- Viša analiza (kurs na doktorskim studijama PMF)
- Harmonijske funkcije, doktorski kurs, Prirodno-Matematički fakultet Sarajevo (2014),

Mentorstva na doktorskim disertacijama
2013. Marijan Marković (Beogradski univerzitet).
2014. Djordjije Vujadinović (Beogradski univerzitet)
Mentorski na magistarskim tezama
2010. Djordjije Vujadinović (UCG)

C). NAUČNO-ISTRAŽIVAČKI INTERES

Geometrijska teorija funkcija: Harmonijske funkcije, Kvazikonformna preslikavanja, Holomorne funkcije, Funkcijonalni prostori: Hardijevi i Bergmanovi prostori, Parcijalne diferencijalne jednačine: Poissonova, Laplaceova, Elliptičke PDE, Diferencijalna geometrija: Harmonijske površi, Minimalne površi; Izoperimetrijska nejednakost itd.

- Upravljanje projektima

1. Rukovodilac nacijonalnog projekta Analiza na mnogostrukosti i primjene (2011-2015), koga finansira Ministarstvo nauke Republike Crne Gore. Projektni tim čine renomirani matematičari iz Crne Gore. Pri tome je projekat pri evaluaciji osvojio maksimalan broj poena od strane međunarodnih eksperata. (Nagrada Ministarstva nauke za najbolji naučni projekat za 2013 godinu)

3. Trenutno je rukovodilac dva bilateralna projekta jednog sa Kinom i drugog sa Hrvatskom.

2. Bio je Rukovodilac uspješnog nacijonalnog projekta Analiza na mnogostrukosti (2008-2011).

- Izvod iz bibliografije

Publikovani (ukupno 65 radova), između ostalog, u sledećim vrhunskim matematičkim časopisima: Advances in Mathematics, Transactions of American Mathematical Society, Calculus of Variations and PDE, International mathematics research notices, Proceedings of American Mathematical Society, Journal D' Analyse Math, Israel Journal of Math, Mathematische Zeitschrift, Annali della Scuola Normale Superiore di Pisa- Classe di Scienze, Annales Academia Scientiarium Fennicae Mathematica, Annali di Matematica Pura ed Applicata, Pacific Journal of Mathematics.
U pripremi imaja još 6 radova koji se nalaze na arxiv.org serveru.

Ukupan broj radova publikovanih na žurnalima koji pripadaju SCI listi je 60. Svoje radove je izložio na više od 25 naučnih konferencija i seminara i to u sljedeće države SAD, Rusija, Japan, Kina, Južna Korea, Njemačka, Francuska, Finska, Rumunija, Srbija, Norveška, Češka, Poljska itd. Njegovi koautori su između ostalog: Noam Elkies, Eero Saksman, Matti Vuorinen, Miroslav Pavlović, Miodrag Mateljević koji su dali svojevrstan pečat modernoj matematici. Kao dokaz ove teze je činjenica da je Noam Elkies svojevremeno postao najmladji profesor u historiji Harvard univerziteta (http://en.wikipedia.org/wiki/Noam_Elkies), dok je Eero Saksman urednik Acta Mathematica"), koji je najprestižniji svjetski matematički časopis (http://www.springer.com/mathematics/journal/11511?detailsPage=editorialBoard Kalajevi radovi su citirani više od 550 puta (www.googlescholar.com). (Spisak radova i konferencija su dati u prilogu).

D) Udžbenici

1. D. Kalaj: Zbirka zadataka iz kompleksne analize, Univerzitet Crne Gore, 2006, 219 str.
2. M. Jaćimović, D. Kalaj: Uvod u kompleksnu analizu, Univerzitet Crne Gore, 2009, 347 str.

Prevodi i adaptacija školskih udžbenika

Prevod i adaptacija udžbenika iz matematike za ukupno 8 razreda za osnovnu i srednju školu sa srpskog (crnogorskog) na albanski jezik u izdanju izdavačke kuće "Zavod za udžbenike i nastavna sredstva" u periodu 2008-2010 i 2014.

E) Uredništva

Urednik je sljedećih matematičkih časopisa:

1. World scientific journal (http://www.hindawi.com/journals/tswj/editors/mathematical.analysis/)
2. Abstract and applied mathematics, http://www.hindawi.com/journals/aaa/psi/
3. Bulletin of mathematical analysis and application, http://91.187.98.171/bmathaa/
4. Albanian journal of mathematics

E) Recenzije i ekspertize:

Recenzija radova za renomirane časopise:
Transaction of AMS, Indiana Journal of mathematics, Proceeding of AMS, Annales Academiæ Scientiarum Fennicæ Mathematica, Applied Mathematics Letters, Abstract and Applied Analysis, Applied Mathematics and Computation, Complex variables and Elliptic equations, Filomat, Publications d'Institute mathematique, Bulletin of the Malaysian Mathematical Sciences Society, Journal of mathematical analysis and application, Bulletin of mathematical analysis and application, Publicationes Mathematicae Debrecen, Worlds scientific journal, Acta Mathematica

Sinica, Turkish journal of math, Mathematica slovaca, Bulletin of London math society, Journal of Indian Academy of Mathematics etc.

Međunarodne ekspertize za projekte:

- Evaluator za projekte iz oblasti matematika koje je raspisalo Israel science fondation države Israel na period 2000-2016.
- Evaluator za projekte iz oblasti matematika koje je raspisalo Israel science fondation države Israel na period 2012-2015.
- Evaluator za projekte iz oblasti matematika koje je raspisalo Israel science fondation države Israel na period 2015-2018.
- Evaluator za projekte iz oblasti matematika koje je raspisalo Ministarstvo prosvjete i nauke Republike Srbije na period 2011-2014
- Evaluator za projekte FONDECYT, Chile 2014.

David Kalaj – Curriculum Vitae

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Education

March 2002 University of Belgrade, Faculty of Mathematics
PhD in Mathematics, Thesis title: "Harmonic Mappings and Quasi-conformal Harmonic Mappings between Convex Domains".

1995 – 1998 University of Belgrade, Faculty of Mathematics
M.SC. Program, Thesis title: "Harmonic diffeomophisms and quasiconformal mappings"
GPA: 10,00/10,00

1991 – 1995 University of Montenegro, Faculty of Sciences
B.SC. in Mathematics
GPA: 9.52/10.00

Fellowships and awards

1993 Annual Fellowship of the Ministry of Education of the Republic of Montenegro

1994 "Decembarska nagrada grada Podgorice" (the Award of the Municipality of Podgorica for distinctive results achieved as a student)

2012 The award for the best project funded by the Ministry of science of Montenegro

Teaching experience

1995 – 1997 Teach. assistant, University of Montenegro, Faculty of Sciences
Mathematical Analysis 2, undergraduate course
Differential Calculus, undergraduate course

1998 – 2002 Teach. assistant, University of Montenegro, Faculty of Sciences
Mathematical Analysis 2, undergraduate course
Complex Analysis, undergraduate course

2002 – 2007 Assist. professor, University of Montenegro, Faculty of Sciences
Complex Analysis, undergraduate course
Mathematical Analysis 3, undergraduate course

2007 – 2012 Associate professor, University of Montenegro
Complex Analysis, Mathematics 1, Mathematics 2,

Mathematics 1, Mathematics 4, (Study programme for school teachers in Albanian Language) undergraduate course.
Mathematical Analysis 3, undergraduate course,
Real and Complex Analysis, graduate course

2012 – Full professor, University of Montenegro

Mentorstva na doktorskim disertacijama

2013, Marijan Markovic (Beogradski univerzitet)
2014, Djordjije Vujadinovic (Beogradski univerzitet)

Mentorska na magistarskim tezama

2010, Djordjije Vujadinovic

DODATNE INFORMACIJE

Born: December 11, 1971, Podgorica, Yugoslavia
Citizenship: Montenegrin
Languages: Albanian (native command), Serbian (native command), English (fluent), Russian (passive), Italian (passive).
Computer skills: Latex, C++, Mathematica software
Projects: a) Establishment and management of Study programme for teacher education at Albanian since 2004.
b) PI of the national project Analysis on manifolds (2008-2011).
c) PI of the national project Analysis on manifolds and applications (2012-2015)

RADOVI

1. D. Kalaj, *Univalent harmonic mappings between Jordan domains*, Publ. Inst. Math., Nouv. Ser. 69(83), 108-112 (2001).
2. D. Kalaj, *On the Nitsche's conjecture for harmonic mappings* Mathematica Montisnigri Vol XIV (2001) 89-94.
3. D. Kalaj, *The Jacobian of harmonic function and of its boundary values*, Revue Roumaine De Mathematiques Pures Et Appliquees Tome XLVII, N 5-6, (2003).
4. D. Kalaj, *On harmonic diffeomorphisms of the unit disc onto a convex domain*, Complex Var. Theory Appl. 48, No.2, 175-187 (2003).
5. D. Kalaj, *Quasiconformal harmonic functions between convex domains*, Publ. Inst. Math., Nouv. Ser. 76(90), 3-20 (2004).
6. D. Kalaj, *On the Nitsche's conjecture for harmonic mappings in R^2 and in R^3*, Publ. Inst. Math. (Beograd) (N.S.) 75(89) (2004), 139–146.
7. D. Kalaj, M. Pavlovic, *Boundary correspondence under harmonic quasiconformal homeomorphisms of a half-plane* Ann. Acad. Sci. Fenn. Math. 30 (2005), no. 1, 159–165.
8. D. Kalaj, *On the Nitsche conjecture for harmonic mappings in R^2 and in R^3*. Israel J. Math. 150 (2005) 241-251.
9. D. Kalaj, M Mateljevic: *Inner estimate and quasiconformal harmonic maps between smooth domains*, J. Anal. Math. 100 (2006), 117-132.
10. Sh. Najafzadeh, S. R. Kulkarni and D. Kalaj *Application of convolution and [illegible] [illegible] on analytic and p-valent functions*, Filomat 20:2 (2006), 115–124.
11. D. Kalaj, [illegible] *[illegible] of PDE [illegible] between spherical [illegible]*, J. Math. Anal. Appl. Volume 327, Issue 1, Pages 1-11, (2007).
12. D. Kalaj, [illegible] *[illegible] harmonic mappings between Jordan domains* Math. Z. [illegible] Number 2, 237-252, 2008.

13. D. Kalaj, M Mateljević, *Quasiconformal and harmonic mappings between [illegible]*
Jordan domains, Novi Sad J. of Mathematics, 38 (2) 2008, 147-156.
14. D. Kalaj, *On harmonic quasiconformal self-mappings of the unit ball*, Ann. Acad. Sci.
Fenn. Math. Vol 33, 261-271, (2008).
15. D. Kalaj, *Lipschitz spaces and harmonic mappings*, Ann. Acad. Sci. Fenn. Math.
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16. D. Kalaj, *On quasiregular mappings between smooth domains*, J. Math. Anal. Appl.
2010, 362, issue 1, Pages 58-63.
17. D. Kalaj, M Mateljevic, *Harmonic q.c. self-mapping and Möbius transformations of*
the unit ball B^n, Pacific J. Math. Vol. 247, No. 2, 2010, 389-406.
18. D. Kalaj, *On an integral inequality and application to Poisson equation*, Applied
Mathematics Letters, 23 (2010) 1016–1020.
19. D. Kalaj, *Quasiconformal harmonic mappings and close to convex domains*, Filomat,
Volume 24, Number 1, April 2010, 63—68.
20. D. Kalaj, M Mateljevic, *On absolutely conformal mappings*, Publ. Math. Debrecen.
77/1-2 (2010), 33-38.
21. R. Meštrović, D. Kalaj, *A converse of Minkowski's type inequalities*, Journal of
Inequalities and applications, Volume 2010 (2010), Article ID 461215, 9 pages
doi:10.1155/2010/461215.
22. D. Kalaj, M. Mateljevic, *Quasiconformal harmonic mappings and generalizations*, J.
Analysis, Volume 18 (2010), 239-260.
23. D. Kalaj, M. Pavlovic, *On quasiconformal self-mappings of the unit disk satisfying*
Poisson differential equation, Trans. Amer. Math. Soc. 363 (2011) 4043—4061.
24. D. Kalaj, *Harmonic maps between annuli on Riemann surfaces*, Israel J. Math. 182
(2011), 123—147.
25. D. Kalaj, M Mateljevic, *On quasiconformal harmonic surfaces with rectifiable*
boundary, Complex Anal. Oper. Theory 5, No. 3, 633-646 (2011).
26. D. Kalaj, M Mateljevic, *On certain nonlinear elliptic PDE and quasiconformal mapps*
between Euclidean surfaces, Potential analysis, Volume 34, Number 1, 13-22, DOI:
10.1007/s11118-010-9177-x (10 pages).
27. D. Kalaj, *Harmonic mappings and distance function* Ann. Scuola Norm. Sup. Pisa
Cl. Sci. (5), Vol. X (2011), 669-681.
28. D. Kalaj, *Isoperimetric inequality for the polydisk*, Annali di matematica pura ed
applicata, Volume 190 (2011), Number 2, 355-369.
29. D. Kalaj, *Estimates of gradient and of Jacobian of harmonic mappings defined in the*
unit disk. Proc. Am. Math. Soc. 139, No. 7, 2463-2472 (2011).
30. D. Kalaj, *Invertible harmonic mappings beyond Kneser theorem and quasiconformal*
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77. D. Kalaj: On Quasiconformal harmonic function of the unit disk onto a convex domain, Roni-Finn Seminar, 2001, Brasov, Rumunija
78. D. Kalaj: 5 International symposium of mathematical analysis and its applications, MAAS, Niška Banja, October 2-6, 2002
79. D. Kalaj: On the Nitsche's conjecture for harmonic mappings in R2 and in R3. 986 TH AMS Meeting, Courant Institute New York, April 12-13, 2003 page 45-46
80. D Kalaj, M Pavlović: Boundary correspondence under harmonic quasiconformal mapping of the halfplane. The book of abstract of XI Congress of Yugoslav Mathematicians page 43, Petrovac, October 2004.
81. D. Kalaj: On the first and on the radial derivative of harmonic function defined on the unit ball. Proceedings of the Workshop devoted to 25 anniversary of the Faculty of Natural Sciences and mathematics, University of Montenegro September 2005.

82. D. Kalaj: On the univalent solution of PDE \Delta u=f between spherical annuli, The book of abstracts of Harmonic Analysis and partial Differential Equations, June 27-July 1, 2005, Kiel, Germany.
83. D. Kalaj: Harmonic and quasiconformal maps, Extremal Problems in Complex and Real Analysis, Peoples Friendship University of Russia, Moscow, Russia May 22-26, 2007. The book of abstracts.
84. D. Kalaj: Quasiconformal harmonic maps, Seminar: Mathematical Colloquium, Beograd 11. 05. 2007. http://www.mi.sanu.ac.yu/colloquiums/mathcoll_programs/mathcoll.may2007.htm, Predavanje po pozivu
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86. D. Kalaj: On quasiconformal harmonic mappings, Congress in memory of Adrien Douady, Paris, France Maj, 2008, Poster
87. D. Kalaj: Boundary correspondence under q.c. harmonic mappings between Jordan domains, Mini conference on quasiconformal harmonic mappings, Beograd, Srbija, 2009, septembar, Predavanje po pozivu.

88. D. Kalaj: On quasiconformal mappings and elliptic PDE in the plane, Helsinki seminar on Analysis, October, 2010, predavanje po pozivu.
89. D. Kalaj: On quasiconformal mappings and elliptic PDE in the plane, Turku seminar on Analysis, October, 2010, predavanje po pozivu.
90. D. Kalaj: Deformation of annuli under smallest mean distortion on Riemann surfaces, Workshop on Quasiconformal mappings and Mappings of finite distortion, Prague, September 2011, Predavanje od 30 minuta.
91. D. Kalaj: Deformation of annuli under smallest mean distortion on Riemann surfaces and generalization of J. C. C. Nitsche Conjecture Workshop on Complex Analysis, Belgrade, February 2012. predavanje po pozivu.
92. D. Kalaj: Deformations of Annuli on Riemann surfaces and the generalization of Nitsche conjecture and Quasiconformal harmonic mappings, Predavanje po pozivu, International Conference on Complex Analysis and Related Topics, Romania, Ploiesti 2012 http://imar.ro/RoFinSem2012/conf.php, Romanian Finish seminar.
93. D. Kalaj: Deformations of Annuli on Riemann surfaces and the generalization of Nitsche conjecture, The 6th European Congress of Mathematics, 2012. Poster.

94. David Kalaj: Cauchy transform and Poisson equation, Turku analysis seminar, Finska (4.10. 2012) http://users.utu.fi/ripekl/seminar/index.html
95. David Kalaj, Energy-minimal diffeomorphisms between doubly connected Riemann surfaces, "Conference on Riemann surfaces and Kleinian groups", held in Osaka University, Japan, from January 12 to January 14, 2013.
96. David Kalaj, Quasiconformal harmonic mappings between surfaces, Conference of Geometric function theory, October 2013, Beograd.
97. David Kalaj, Muckenhoupt weights and Lindelöf theorem for harmonic mappings, March 2014, Helsinki seminar of analysis, Helsinki 2014.
98. David Kalaj, Quasiconformal harmonic mappings between surfaces. March 2014, Helsinki seminar of analysis, Helsinki 2014.
99. David Kalaj, Muckenhoupt weights and Lindelöf theorem for harmonic mappings, ICM August 2014, Seoul, Korea, Short communication.
100. David Kalaj, Harmonic and quasiconformal mappings and generalizations, School of Mathematical Science Huaqiao University, P.R.China, January 26, 2015.
101. David Kalaj, On quasi-inversions School of Mathematical Science Huaqiao University, P.R.China, January 27, 2015
102. David Kalaj, Energy-minimal diffeomorphisms between doubly connected Riemann surfaces, School of Mathematical Science Huaqiao University, P.R.China, January 2? 2015.

103. David Kalaj, Poisson equation and Cauchy transform; School of Mathematical Science Huaqiao University, P.R.China, January 28, 2015.
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106. David Kalaj: "Schwarz lemma for harmonic functions, School of Mathematical Science" Huaqiao University, P.R.China, January 27, 2015.
107. David Kalaj, "The J. C. C. Nitsche conjecture and generalization" School of Mathematical Science Huaqiao University, P.R.China, January 25, 2015.

KNJIGE

1. D. Kalaj: Zbirka zadataka iz kompleksne analize, Univerzitet Crne Gore, 2006, 219 str.
2. M. Jaćimović, D. Kalaj: Uvod u kompleksnu analizu, Univerzitet Crne Gore, 2009, 347 str.

PREVODI

Prevod i adaptacija sljedećih udžbenika iz matematike sa srpskog (crnogorskog) na albanski jezik u izdanju izdavačke kuće "Zavod za udžbenike i nastavna sredstva" u periodu 2008-2010.

1) Matematika IV (četvrti razred osnovne škole)
2) Matematika V (peti razred osnovne škole)
3) Matematika VIII (osmi razred osnovne škole)
4) Matematika IX (deveti razred osnovne škole)
5) Matematika II (drugi razred srednje škole)
6) Matematika III (treći razred srednje škole).
7) Algoritmi i programiranje (treći i četvrti razred srednje škole).

Citati: 550 citata (http://scholar.google.com).

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Универзитет у Београду-Математички факултет издаје следећу

П О Т В Р Д У

Овим се потврђује да је академик Миодраг Матељевић изабран у звање редовног професора 29.03.1995. године.

Потврда се издаје на лични захтев именованог, ради чланства у комисији за оцену докторске дисертације кандидата Антона Гјокаја.

ДЕКАН
УНИВЕРЗИТЕТА У БЕОГРАДУ-
МАТЕМАТИЧКОГ ФАКУЛТЕТА

Проф. др Зоран Ракић

CURRICULUM VITAE OF PROFESSOR M. MATELJEVIĆA

Year and Place of birth	:	1949; Valjevo (Serbia)
Bachelor degree	:	Mathematics group PMF (Faculty of Natural Science), Belgrade 1973
Master thesis	:	Inequalities in H^p and their extremal properties (in serbian: Nejednakosti u H^p prostorima i njihova ekstremalna svojstva), Magistarski rad, 1976
Doctoral thesis	:	Estimates of norms and extremal problems in H^1 (in serbian: " Ocene normi i ekstremalni problemi u H^1"), 1979, PMF in Belgrade

POSITIONS HELD:

- Assistant at PMF (1973-83)
- Docent(assistant professor) 1983-89
- Associate professor 1989-95
- Full professor 1995 onwards
- Chairmen of Department of Complex Analysis since 1995.
- Head of project Mathematics, Mechanics and Computer sciences (in serbian "Matematika, Mehanika i Računarstvo") since 1995.
- Vice-dean at Department of Mathematics 1995.
- Founder and chief of Seminar for Complex Analysis (GFT) since 1991.
- Dean at Department of Mathematics (Faculty of Mathematics) 2007- 2014.

SCIENTIFIC RECOGNITION:

- The City of Belgrade Science Award for 2006.
- Award " prof. dr Vojislav Stojanović", Union of university professors and scientists of Serbia, 2010
- Editor (member of editorial board) for several science journals: "Filomat" (2009-), "Applicable Analysis and Discrete Mathematics" (2008-), "The Journal of Analysis" and "Publications de l'Institut Mathematique" (2011-).
- Corresponding member of Serbian Academy of Sciences & Arts from 2012- 2018.
- Academician, Full member of Serbian Academy of Sciences & Arts from 2018- .

VISITING POSITIONS HELD:

- Winter semester 1981 at university Winsconsin-Madison.
- Winter semester 1988 as associate professor at University of Pittsburgh

- Associate Professorship during 1988/89. at Wayne State University, Detroit.

PUBLICATIONS:

- He published about 140 scientific papers in well internationally known journals including Journal d'Analyse, Studia mathematica, Pacific Journal of Mathematics, Mich. Math. J., J. Math. Anal. Appl., Proc. Amer. Math. Soc., Proc. Edinb. Math. Soc., J. Math. Chem., Lect. Notes Math., Ann. Acad. Sci. Fenn. Math., Potential Analysis, MATCH, etc.

  He delivered about 30 invited lectures and about 10 plenary lectures: IWWA USA, Nevannlina Colloquium Switzerland, 4 lectures at Finish -Rumanian seminar, Oberwolfach, Grotzsch conference, invited by Polish and Bulgarian academy of science,...; His paper are quoted about 1670 times.

  The results with proofs, concerning Teichmuller problem, Gehring's problem, the isoperimetric inequality, harmonic and analytic functions, are cited in known Monograph;see,for example, p.100,

- Lectures on Quasiconformal Mappings: Second Edition Lars V. Ahlfors with additional chapters by C. J. Earle and I. Kra, M. Shishikura, J. H. Hubbard, University Lecture Series 2006; 162 pp; Volume 38,

  and

- E Reich, Extremal Quasiconformal Mappings of the Disk, Handbook of Complex Analysis: Geometric Function Theory, Volume 1, edited by Kuhnau, 2002 Elsevier Science B.V.

Among the other things, he has solved Sinai's problem (Sinai is awarded with Abel prize) and he has showed that HQC mapping between Lyapunov domains are bi-lipshitz (well known problem in HQC-theory) recently.

TEACHING RESPONSIBILITIES:

- Courses taught include Mathematica I & II, Analysis I & II, Theory of Real and Complex functions, Complex Analysis
- Graduated courses taught include: Analysis on manifolds (USA), Complex Analysis(USA), Conformal invariant, Quasiconformal mappings, Complex dynamics, etc.

INVITED LECTURE SERIES:

1. In the Chinese University of Hong Kong, Hong Kong, during Oct., 2001
2. In Warwick during 30 Nov - 30 Dec., 2003
3. In Warwick during 5 Dec - 20-Dec., 2004
4. Special course in "Quasiconformal mapping and Teichmuller spaces" na Scola Normala Superioara, Buchurest, 2004.
5. In the University of Helsinki, october 2005
6. Several plenary lectures at Helsinki-Turku Seminar, october 2005.
7. Five invited lecture at INTERNATIONAL WORKSHOP ON HARMONIC MAPPINGS AND HYPERBOLIC METRICS, (IWHMHM09), DECEMBER 10 - 19, 2009

8. Two invited lecture at HQM2010 Invited Speakers, ICM 2010 Satellite Conference, International Workshop on Harmonic and Quasiconformal Mappings, (HQM2010), IIT Madras, August 09-17, 2010.

9. Plenary one-hour lecture at the eighth international conference on Computational Methods and Function Theory (CMFT 2017), July 10-15, 2017.

DESCRIPTION ON CONTRIBUTION FROM BOOKS AND MONOGRAPHS:

- Notable books:

  1. M. Mateljević, M.Jevtić, Analitičke Funkcije, zbirka, Beograd, 1986

  2. Book - monograph Kompleksne funkcije 1 & 2, Beograd 2006, Društvo Matematičara Srbije

  3. M. Mateljević, Kompleksna Analiza 1, Zavod za udžbenike, Beograd 2012.

  4. Book - monograph, Monograph, M. Mateljević, Kompleksna Analiza 2, Zavod za udžbenike, Beograd 2012.

  5. M. Mateljević, Topics in Conformal, Quasiconformal and Harmonic maps, Zavod za udžbenike, Beograd 2012

- The results with proofs, concerning Teichmuller problem, Gehring's problem, the isoperimetric inequality, harmonic and analytic functions, are cited in known Monograph.

Short biography of M. Mateljević

Dean at Department of Mathematics (Faculty of Mathematics) 2007-2014. Corresponding member of Serbian Academy of Sciences & Arts from 2012-2018. Academician of Serbian Academy of Sciences & Arts from 2018 (Nov 8). Winter semester 1988 as associate professor at University of Pittsburgh. Associate Professorship during 1988/89. at Wayne State University, Detroit. The winner of the City of Belgrade Science Award for 2006. His research is related to harmonic maps, quasiconformal mappings, geometric inequalities and elliptic partial differential equation. He published about 130 scientific papers in well internationally known journals, which are quoted about 1670 times, and notable books Topics in Conformal, Quasiconformal and Harmonic maps 2012 and Kompleksne funkcije 1 & 2, 2006. He delivered about 50 invited lectures and about 20 plenary lectures: IWWA USA, Nevannlina Colloquium Switzerland, 4 lectures at Finish -Rumanian seminar, Oberwolfach, Grotzsch conference, Computational Methods and Function Theory (CMFT 2017),invited by Polish and Bulgarian academy of science,...;

List of publications, Mateljević, M., 3/19/2023 133 rada

List A

1. Mateljević, M., *On linked Jordan curves in R^3*, Mat. Vesnik 12 (27) (1975), 285-286.
2. Mateljević, M., *The isoperimetric inequality and some extremal problems in H^1*, Lect. Notes Math. 798 (1980), 364-369.
3. Mateljević, M., Pavlović, M., *On the integral means of derivates of the atomic function*, Proc. Amer. Math. Soc. Vol. 86 No.3 (1982), 455-458.
4. Mateljević, M., Pavlović, M., *Behavior of power series with positive coefficients and Hardy Spaces*, Proc. Amer. Math. Soc. Vol. 87 No.2 (1983), 309-316.
5. Mateljević, M., Pavlović, M., *L^p-behaviour of the integral means of analytic functions*, Studia Mathematica Vol. 77 (1983), 219-237.
6. Mateljević, M., Pavlović, M., *New proofs of the isoperimetric inequality and some generalizations*, J. Math. Anal. Appl. 98 (1984), 25-30.
7. Mateljević, M., Pavlović, M., *Multipliers of H^1 and BMOA*, Pacific Journal of Mathematics 146 (1990), 71-84.
8. Mateljević, M., *An extensions of the area theorem*, Complex Variables 15 (1990), 155-157.
9. Mateljević, M., Pavlović, M., *Some inequalitics of isometric type concerning analytic and subharmonic functions*, Publ. Inst. Math. Belgrade 50 (69) (1991), 123-130.
10. Mateljević, M., Pavlović, M., *An extension of the Forelli-Rudin theorem*, Proc. Edinb. Math. Soc. 2 36 (1993) No.3, 375-389.
11. Mateljević, M., *Note on Schwarz lemma, curvature and distance*, Coll. Sci. papers of the Faculty of Science Kragujevac 16 (1994), 47 – 51, Zbornik konferencije.
12. Mateljević, M., *Dual of the Bergman space defined on a hyperbolic plane domain*, Publ. Inst. Math. Belgrade 56 (70) (1994), 135-139.
13. Mateljević, M., Pavlović, M., *The best approximation and composition with inner functions*, Mich. Math. J. 42 (1995), 367-378.
14. Mateljević, M., *Estimates for gradient, BMO and Lindel of theorem*, Publ. Inst. Math. Belgrade 58 (72) (1995),162-166.
15. Mateljević, M., Marković, V., *The unique extremal QC mapping and uniqueness of Hahn-Banach extensions*, Mat. Vesnik 48 (1996), 107-112.
16. Božin, V., Marković, M., Mateljević, M., *The unique extremality in the tangent space of Teichmuller space*, Integral Transforms and Special Functions 6 (1997), 223-227.
17. Marković, V., Mateljević, M., *New version of Grotzsch principle and Reich-Strebel inequality*, Mat. Vesnik 49 (1997), 235-239.

18. Božin, V., Lakić, N., Marković, V., Mateljević, M., *The unique extremality*, Journal d'Analyse 75 (1998), 299-338.
19. M. Mateljević, V. Marković, *New versions of the main inequality and uniqueness of harmonic maps*, J. d' Analyse Math 79, 1999, 315-334.
20. Anić, I., Marković, V., Mateljević, M., *Uniformly Bounded Maximal Φ-disks, Bers Spaces and Harmonic Maps*, Proc. Amer. Math. Soc., Vol 128, No 10, 2000, pp 2947-2956.
21. M. Mateljević, *The unique extremality II*, Mathematical Reports Vol. 2 (52) No. 4, 2000, 503-525, plenarno predavanje na: The VIII th Romanian-Finish Seminar, Iassy, 23-27 Avg 1999.
22. I. Anić, M. Mateljević, D. Šarić, *Extremal metric and modules*, Czechoslovak Math. J., 2002, Vol 52 (127), No 2, 225-235.
23. M. Mateljević, *A version of Bloch theorem for quasiregular harmonic mappings* Rev Roum Math Pures Appliq 47 (2002/2003) 5-6, pp 705-707, ISSN 0035-3965, plenarno predavanje na: The IX th Romanian-Finish Seminar, Brasov, 27-31 Avg, 2001.
24. M. Mateljević, *Estimates of the modulus of derivates of harmonic univalent mappings*, Rev Roum Math Pures Appliq 47 (2002/2003) 5-6, 709-711, plenarno predavanje na: The IX th Romanian-Finish Seminar, Brasov, 27-31 Avg 2001; i na Funktionetheorie, 11-17 Feb 2001, p. 7, Mathematisches Forschungsinsitut Oberwolfach.
25. M. Mateljević, *Ahlfors-Schwarz lemma and curvature*, Kragujevac Journal of Mathematics (Zbornik radova PMF), Vol. 25, 2003, 155-164, Zbornik konferencije.
26. M. Mateljević, *Dirichlet's principle, distortion and related problems for harmonic mappings*, Publication l'Inst Math - Belgrade, nouvelle serie 75 (89), 2004, 147-171 (special number Quasiconformal and Harmonic mappings, special guest editor M. Mateljević)
27. M. Mateljević, *Dirichlet's principle, uniqueness of harmonic maps and extremal qc mappings*, Zbornik radova 10 (18), Two topics in mathematics, editor B. Stanković, 2004, 41-91.
28. I. Gutman, M. Mateljević, *Note on the Coulson integral formula*, J. Math. Chemistry, Vol 39 (2006), No 2, 259-266.
29. Kalaj, D., Mateljević, M., *Inner estimate and quasiconformal harmonic maps between smooth domains*, J. d'Analyse Math. Vol. 100 (2006), 117-132.
30. M. Mateljević, *Distortion of harmonic functions and harmonic quasiconformal quasi-isometry*, Revue Roum. Math. Pures Appl. Vol. 51 (2006), 5-6, 711-722, plenary lecture; http://www.imar.ro
31. M. Knežević, M. Mateljević, *On the quasi-isometries of harmonic quasi-conformal mappings*, J Math Anal Appl, 2007, Vol 334, No 1, 404-413.
32. M. Mateljević, *Quasiconformal and Quasiregular harmonic analogues of Koebe's Theorem and Applications*, Ann. Acad. Sci. Fenn.-M, Vol 32, (2007), No 2, 301-315.
33. M. Mateljević, I. Gutman, *Note on the Coulson and Coulson-Jacobs integral formulas*, MATCH Commun. Math. Comput. Chem. Volume 59, 2008, No 2, 257-268.

34. M. Mateljević, M. Arsenović and V. Kojić, *On Lipschitz continuity of harmonic quasiregular maps on the unit ball in R^n*, Ann. Acad. Sci. Fenn., Vol. 33, No 1, 2008, 315-318.
35. M. Mateljević, *Versions of Koebe 1/4 theorem for analytic and quasiregular harmonic functions and applications*, Publications de l'institut Mathematique, Nouvelle serie, tome 84 (98) (2008), 61-72.
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МИХАИЛО ПЕТРОВИЋ АЛАС ЖИВОТ, ДЕЛО, ВРЕМЕ, ПОВОДОМ 150 ГОДИНА ОД РОЂЕЊА

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Na osnovu člana 72 stav 2 Zakona o visokom obrazovanju („Službeni list Crne Gore" br. 44/14, 52/14, 47/15, 40/16, 42/17, 71/17, 55/18, 3/19, 17/19, 47/19, 72/19, 74/20 i 104/21) i člana 32 stav 1 tačka 9 Statuta Univerziteta Crne Gore, Senat Univerziteta Crne Gore na sjednici održanoj 15.12.2021. godine, donio je

O D L U K U
O IZBORU U ZVANJE

Dr Marijan Marković bira se u akademsko zvanje vanredni profesor Univerziteta Crne Gore za **oblast Matematika** na Prirodno-matematičkom fakultetu Univerziteta Crne Gore, na period od pet godina.

SENAT UNIVERZITETA CRNE GORE
PREDSJEDNIK

Prof. dr Vladimir Božović, rektor

BIOGRAFIJA I BIBLIOGRAFIJA – MARIJAN MARKOVIĆ

Biografija

Marijan Marković je rođen 21. aprila 1982. g. u Kotoru. Završio je gimnaziju *Slobodan Škerović* u Podgorici 2001. g. Potom je 2005. g. diplomirao na Prirodno-matematičkom fakultetu UCG.

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ODLUKU
O IZBORU U ZVANJE

Dr Đorđije Vujadinović bira se u akademsko zvanje vanredni profesor Univerziteta Crne Gore za **oblast Matematika**, na Prirodno-matematičkom fakultetu Univerziteta Crne Gore, na period od pet godina.

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Đorđije Vujadinović je rođen 1985. u Pljevljima. Osnovnu školu je završio u Ulcinju i Podgorici, a gimnaziju "Slobodan Škerović" u Podgorici. Osnovne studije iz oblasti matematike završio je na Prirodno-matematičkom fakultetu u Podgorici, 2004. godine. Poslijediplomske studije iz oblasti Matematičke analize je završio na istom fakultetu, gdje je i odbranio magistarsku tezu pod nazivom "Integracija na apstraktnim prostorima i maksimalna funkcija" aprila 2010. godine.

Doktorske studije iz oblasti Matematičke analize je započeo 2009. godine na Matematičkom fakultet u Beogradu, Univerzitet u Beogradu. Doktorsku disertaciju pod nazivom "Ocjene norme integralnih operatora na prostorima Besova i Bloha " odbranio je 21.11.2014.

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PODACI O RADNIM MJESTIMA I IZBORIMA U ZVANJE

Đorđije Vujadinović je kao saradnik u nastavi bio u radnom odnosu na Univerzitetu Crne Gore od 2008. do 2015. godine, kada odlazi na studijski boravak u Rimu.
Početkom februara 2016. je izabran u zvanje docenta na Univerzitetu Crne Gore.
Odlukom Senata Univerziteta Crne Gore 08.04.2021.godine Đorđije Vujadinović izabran je u zvanje vandrednog profesora na Prirodno-matematičkom fakultetu u Podgorici.

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